Thermal Physics I, test 2

31 Oct 2025, 18.15-20.15

Write your name and student number on every sheet

Extra-time students: 10 minutes per hour => 20 min extra

Problem 1. Preliminaries [10 pts]

a) Provide a short yet accurate statement of the Second Law of Thermodynamics. [5 pts]

b) Derive the Maxwell relation [5 pts]

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

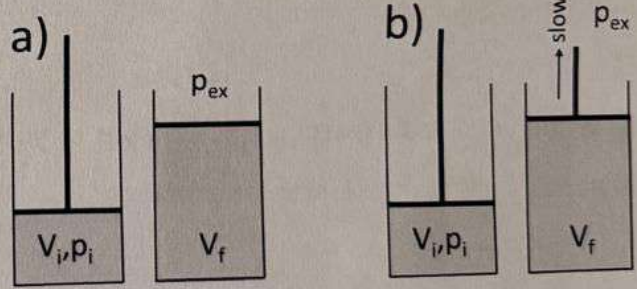
Problem 2. Carnot Cycle [15 pts]

a) Draw the Carnot cycle in a *p, V* diagram, clearly indicate and label the different steps, and the direction of heat flow (if any). [10 pts]

b) How much entropy is produced in the Carnot cycle? Explain your answer. [5 pts]

Problem 3. Expansion work and reversibility [30 pt]

a) A cylinder is filled with 1 mole of an ideal gas (T =25°C, p=14 bar). Initially, the gas is compressed by a massless piston that can move without friction once it is released from its initial position. The ideal gas will subsequently be expanded to an external pressure of 1 bar and to a volume of 23.5 liter. The piston is unlocked and the gas is allowed to expand freely until it reaches the external pressure. Calculate the expansion work (8 pt) .



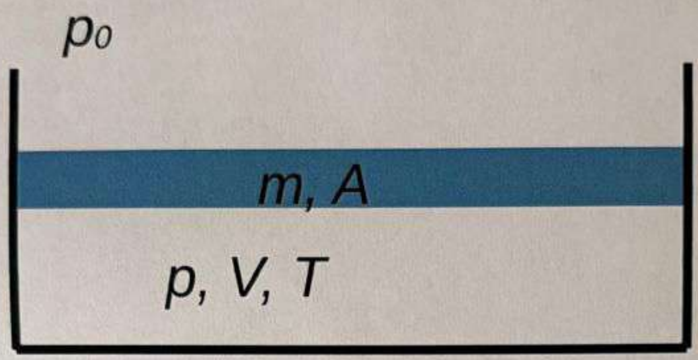
b) Start from the same initial situation as in a). Now the piston is unlocked and the gas is only allowed to expand very slowly until it reaches the external pressure of p = 1 bar. The process is isothermal. Calculate the expansion work. (8 pt).

c) Draw a *p*-*V* diagram for both processes and indicate the expansion work. (8 pt).

d) Which process (a or b) can be considered thermodynamically reversible and why? (6 pt).

Problem 4. Adiabatic, reversible compression and expansion of an ideal gas [25 pts]

An ideal gas (*n* moles) is confined to a container that is closed by a frictionless piston of mass *m* and area *A*. The weight of the piston is given by *mg*, where *g* is the gravitational acceleration. The pressure of the atmosphere is denoted by p_0.



a) Derive an expression for the pressure *p* in the container. [5 pts]

b) What is the volume (*V*) of the gas? [5 pts]

c) Next, you press the piston down with a force *f(x)* over a short(!) distance *x*. This changes the volume to $V - \delta V(x)$and the pressure to $p + \delta p(x)$. You may assume that the gas undergoes adiabatic compression. Derive an expression for the force *f(x)* and use that $(1+z)^{\beta} \approx 1 + \beta z$ for $|z| \ll 1$. [10 pts]

d) When you let the piston go, it will start to oscillate and the gas undergoes adiabatic expansion and compression. The resulting oscillation can be described by

$x(t) = B\sin(\omega t)$,

where ω is the angular frequency of the oscillation. Calculate the angular frequency ω. [5 pts]

Please check whether your name and student number are on every sheet